Burst Statistics Using the Lag-Luminosity Relationship

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Abstract. Using the lag-luminosity relation and various BATSE catalogs we create a large catalog of burst redshifts, peak luminosities and emitted energies. These catalogs permit us to evaluate the lag-luminosity relation, and to study the burst energy distribution. We find that this distribution can be described as a power law with an index of $\alpha = 1.76 \pm 0.05$ (95% confidence), close to the $\alpha = 2$ predicted by the original quasi-universal jet model.

INTRODUCTION

Jet models predict the distribution of the isotropic-equivalent energy E_{iso}: quasi-universal jet profile models predict an approximate power law distribution with index $\alpha = 2$, where $N(E_{iso}) \propto E_{iso}^{-\alpha}$[1]. The isotropic-equivalent energy E_{iso} is the total energy radiated if the observed flux were radiated isotropically. To study the distribution of burst intensities, we used the lag-luminosity relationship to create a burst database with redshifts, peak luminosities, and burst energies, and then we fit energy distributions to the burst database. Of course, this database can be used for other studies.

In the lag-luminosity relation[2] the peak bolometric luminosity L_B is a function of the lag τ_B between two energy bands in the burst's frame—$L_B = Q(\tau_B)$. But τ_0 is measured in our frame. We model $\tau_B = (1+z)^c \tau_0$: time dilation contributes -1 to c, while the redshifting of temporal structure with a smaller lag from higher energy contributes ∼1/3 (pulses are narrower at high energy). The peak bolometric luminosity is related to the peak bolometric energy flux $F_B = L_B/[4\pi D_L^2]$, where D_L is the luminosity distance. The peak bolometric energy flux is related to the peak photon flux P (integrated over an energy band, e.g., 50–300 keV for BATSE data): $F_B = \langle E \rangle P$. The result is an implicit equation that must be solved for each burst:

$$P = Q\left((1+z)^c \tau_0\right) / \left[\langle E \rangle 4\pi D_L^2\right] \qquad (1)$$

After solving eq. 1 for the redshift, L_B and E_{iso} can be calculated from F_B and the energy fluence, respectively.

The original lag-luminosity relation was a single power law, e.g., $L_B \propto \tau_B^{-1.15}$. But this power law over-predicts the luminosity of GRB980425 (assuming this burst was SN1998bw). Consequently Salmonson[3] and Norris[4] suggested breaking the single power law; for τ_B >0.35 s the power law index is -4.7. A population of nearby, long lag bursts resulted.

With a database of bursts with E_{iso} we can now calculate the energy distribution. The methodology presented here[5] can also be applied to the luminosity function. The probability of detecting a given energy is truncated by the detection threshold: $p(E_{iso}\,|\,E_{iso,th}M(\vec{a}))$ where $E_{iso,th}$ is the threshold value of E_{iso} for that burst and $M(\vec{a})$ is the model (e.g., the functional form of the energy distribution) with parameters $\vec{a}$. For the ensemble of bursts the probability of detecting bursts with the observed energies is

$$\Lambda = \prod_i p\left(E_{iso,i}\,|\,E_{iso,th,i}M(\vec{a})\right) \tag{2}$$

where the product is over each burst. This probability is the likelihood for the model $M(\vec{a})$. In frequentist statistics, we maximize Λ with respect to the parameters $\vec{a}$ to get a best fit value. In Bayesian statistics the likelihood is a factor in the "posterior," which can be used for confidence ranges and best fit values; the Bayesian approach allows the use of "priors" reflecting our expectations for $\vec{a}$.

Note that to study the energy distribution we do NOT need a complete sample in terms of observed fluences, only a sample that has no bias on the intrinsic E_{iso}. There can be gaps in the distribution of peak fluxes, but E_{iso} has to be drawn uniformly from $p(E_{iso}\,|\,E_{iso,th}M(\vec{a}))$ in our sample. On the other hand, if we want the burst rate per comoving volume as a function of redshift, then we do need a complete sample.

But is the resulting energy distribution a good representation of the data? The likelihood (frequentist approach) or posterior (Bayesian approach) can be used to compare models (functional forms), but do not indicate "goodness-of-fit." However, our methodology assumes the energies are drawn uniformly from $p(E_{iso}\,|\,E_{iso,th}M(\vec{a}))$. The cumulative distribution of $p(E_{iso}\,|\,E_{iso,th}M(\vec{a}))$ should therefore be a straight line, and the average value should be 1/2, with a statistical uncertainty of $[12N]^{-1/2}$ for N bursts.

RESULTS

We started with 1438 BATSE bursts for which we calculated lags. Of these, 1218 have positive lags. These bursts also have hardness ratios, peak fluxes and durations. To calculate the average energy $\langle E \rangle$ we used the "GRB" spectral fits of Mallozzi et al.[6] to the peaks of 580 of these bursts. For the 858 bursts without fits we assumed average spectral indices $\alpha = -0.8$ and $\beta = -2.3$. Plotting HR_{32} (the 100–300 keV to 50–100 keV hardness ratio) vs. E_p shows a clear correlation which can be approximated by E_p=240 HR_{32}^2 keV; we used this relation for the bursts without spectral fits.

Redshifts were calculated for this database for both the original simple power law lag-luminosity relation and the broken power law Salmonson[3] and Norris[4] introduced to incorporate GRB980425. As expected, the difference in the lag-luminosity relations is apparent at low redshifts: the broken power law results in a population of nearby bursts. There were few physically implausible high z bursts (e.g., $z > 20$) and thus no additional cutoffs on the lag-luminosity relation are required. In the absence of additional information, the choice between the two lag-luminosity relations depends on whether GRB980425 is considered to be a typical low luminosity burst. For the remainder of this analysis we use a single power law lag-luminosity relation.

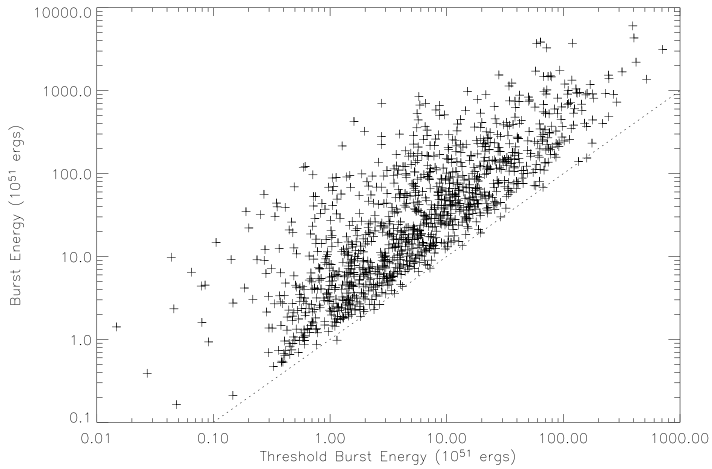


FIGURE 1. Scatter plot of the isotropic equivalent energy E_{iso} vs. the detection threshold.

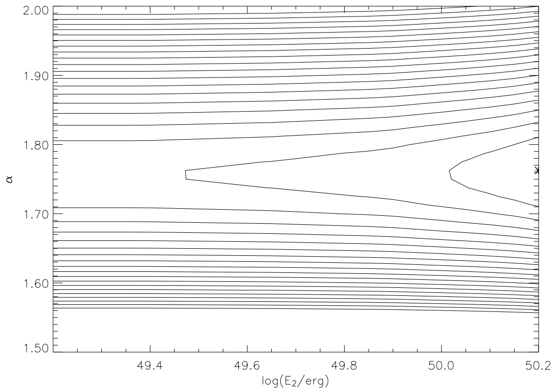
As an aside, we found that the redshift calculation is sensitive to the value of $\langle E \rangle$. Calculating this quantity inconsistently can introduce errors into the resulting database.

We calculated the energy E_{iso} for each burst from the redshift and energy fluence. The results were reasonable (see Fig. 1): few bursts had $E_{iso} > 10^{54}$ erg or $E_{iso} < 10^{51}$ erg.

The energy detection threshold $E_{iso,th}$ can be calculated by scaling E_{iso} by the ratio of the threshold peak photon flux to the observed peak flux. BATSE's threshold peak flux was P_{min} ∼0.3 ph cm^{-2} s^{-1}; however, the number of bursts in our sample with E_{iso} just above the threshold is suspiciously low (see Fig. 1), suggesting that the sample's true threshold was greater than 0.3 ph cm^{-2} s^{-1}. Consequently we used P_{min} ∼0.5 ph cm^{-2} s^{-1} as the threshold, deleting bursts with $P < 0.5$ ph cm^{-2} s^{-1}.

The left hand side of Fig. 2 shows the likelihood surface for our sample assuming a power law functional form, where the two parameters are the low energy cutoff E_2 and the power law index α (i.e., $N(E_{iso}) \propto E_{iso}^{-\alpha}$ for $E_{iso} \geq E_2$). The likelihood is maximized by E_2 equal to the lowest observed value E_{iso}, although lower values are not ruled out. The best fit spectral index is $\alpha = 1.76 \pm 0.05$ (95% confidence). Although $\langle P(> E_{iso})\rangle$= 0.4642±0.0089 ($N$=1054, assuming P_{min} ∼0.5 ph cm^{-2} s^{-1}) deviates from 1/2 by 4σ, considering the possible systematic errors (e.g., in the estimation of E_p from the hardness ratio), this value of $\langle P(> E_{iso})\rangle$ indicates that a power law energy distribution is a fairly good characterization of the data.

We also tried a lognormal energy distribution (right hand side of Fig. 2). The maximum likelihood occurs at $E_{\rm iso,cen} = 3 \times 10^{51}$ ergs and $\sigma_E = 2.7$. The surface's shape indicates that the data permit a high central value of E_{iso} and a narrow distribution, or a low central value of E_{iso} and a broad distribution. The observational cutoff truncates the true energy distribution, and the low energy extent is relatively unknown. We find $\langle P(> E)\rangle$=0.4821±0.0089 (N=1054, assuming P_{min} ∼0.5 ph cm^{-2} s^{-1}), consistent with 1/2 at the 2σ level.



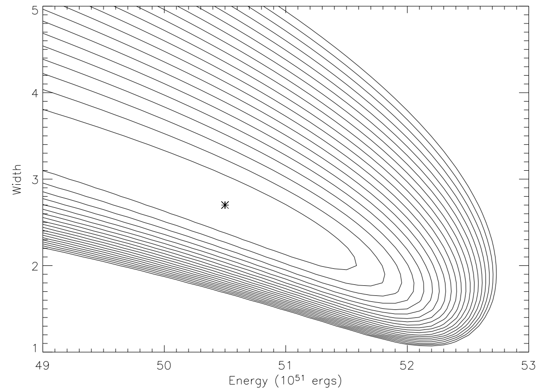


FIGURE 2. Contour plots of the likelihood surface for a power law energy distribution (left) and lognormal energy distribution (right). The power law has a low energy cutoff E_2 and power law index α; the contours are spaced by Δlog(likelihood)=1. The lognormal distribution has a central value $E_{\rm iso,cen}$ and a logarithmic width σ_E; the contours are spaced by Δlog(likelihood)=10.

IMPLICATIONS

A quasi-universal jet profile that is a power law in the off-axis angle θ—the energy per solid angle $\varepsilon(\theta) \propto \theta^k$—results in a power law energy distribution (or luminosity function) with index $\alpha = 1 - 2/k$ (hence $\alpha = 2$ for $k = -2$), while a Gaussian profile results in $\alpha = 1$. Lloyd-Ronning et al.[1] found that if the profile parameters are distributions, the luminosity functions could be approximated by power laws with $\alpha \sim 2$ for power law profiles and $\alpha \sim 1$ for Gaussian profiles, but with curvature. The additional degrees of freedom introduced by varying the parameters give the jet models the freedom to fit a wide variety of energy distribution shapes. We find that our burst data can be fit by a power law energy distribution with $\alpha = 1.76 \pm 0.05$ (95% confidence); considering only the statistical uncertainty the power law distribution is formally not a good fit, but with the likely systematic uncertainties the power law distribution is probably a good description of the data. While our power law fit is inconsistent with the original jet profile model ($k = -2$ and therefore $\alpha = 2$), it is consistent with the jet profile models where parameters are permitted to vary.

A log-normal energy distribution also describes the data; the data permit a smaller average energy if the distribution is wider.

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